2017

(April)

STATISTICS

(Elective/Honours)

(Probability Distributions and Statistical Inference)

[STEH–2(TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Derive Poisson distribution as the limiting case of binomial distribution stating clearly the assumptions on which it is based. 5+1=6

   (b) Find the *(i)* moment-generating function and *(ii)* cumulant-generating function for discrete random variable X following the geometric distribution : 3+3=6

$$P(X=r)=(1-p)\,p^{r-1};\ r=1,\ 2,\ \ldots$$

2. (a) Find the moment-generating function of trinomial distributions and hence find its mean and variance. 3+3=6

(b) Using moment-generating function, what is the distribution of $Y = n - X$, if X is binomially distributed with parameters n and p? 2

(c) If X and Y are independent Poisson variates, such that

$$P(X=1) = P(X=2) \text{ and } P(Y=2) = P(Y=3)$$

find the variance of $X - 2Y$. 4

UNIT—II

3. (a) Show that a linear combination of independent normal variates is also a normal variate. 4

(b) If the first two cumulants of normal density function are 2 and 3 respectively, then write down the normal probability function. 2

(c) If $X \sim \exp(\lambda)$, then find the value of x_a such that $P[X > x_a] / P[X \le x_a] = a$. 3

(d) Find the moment-generating function of rectangular distribution over the interval $[\alpha, \beta]$. 2

4. (a) For a bivariate normal distribution

$$f_{XY}(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}(x^2 - 2\rho xy + y^2)\right\}$$

$$-\infty < (x, y) < \infty$$

find the marginal distributions of X and Y. 5

(b) Write short notes on : 3+3=6

(i) Box plot

(ii) Q-Q plot

UNIT—III

5. (a) What do you mean by sampling distribution of a statistic? Obtain the sampling distribution of sample sum from a Poisson distribution. 2+4=6

(b) Define Fisher's t-statistic and write the p.d.f. of Students t-distribution with n degrees of freedom. 2

(c) If X is chi-square variate with n d.f., then prove that for large n

$$\sqrt{2X} \sim N(\sqrt{2n}, 1)$$ 3

6. (a) State weak law of large numbers. Examine whether the weak law of large numbers holds for the sequence $\{X_k\}$ of

independent random variables defined as follows : 2+4=6

$$P(X_k = \pm 2^k) = 2^{-(2k+1)}$$

$$P(X_k = 0) = 1 - 2^{-2k}$$

(b) For geometric distribution $p(x) = 2^{-x}$; $x = 1, 2, 3, \ldots$, prove that Chebyshev's inequality gives

$$P[|X-2| \le 2] > \frac{1}{2}$$

while the actual probability is $\frac{15}{16}$. 5

UNIT—IV

7. *(a)* Distinguish between point estimation and interval estimation. 3

(b) What properties are being usually held by maximum likelihood estimators? 4

(c) Describe the method of moments for estimating the parameters. 4

8. *(a)* Define minimum variance unbiased estimator. If T_1 is an MVUE of $\gamma(\theta)$ and T_2 is any other unbiased estimator of $\gamma(\theta)$ with efficiency $e < 1$, then show that no unbiased linear combination of T_1 and T_2 can be an MVUE of $\gamma(\theta)$. 1+5=6

(b) Obtain $100(1-\alpha)\%$ confidence intervals for the parameters *(i)* μ and *(ii)* σ^2 of the normal distribution

$$f(x; \mu, \sigma) > \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \ -\infty < x < \infty$$ 5

UNIT—V

9. *(a)* Define type I and type II errors. Which error is more harmful? 2+1=3

(b) Obtain the test statistic for testing the significance for single mean, in random sampling from a large population. State the hypothesis and the distribution of the test statistic. 3

(c) Write a note on the chi-square test of goodness of fit of a random sample to a hypothetical distribution. 5

10. *(a)* Explain the large sample test for testing the significance of difference between two population proportions. 4

(b) Explain paired *t*-test for significance of difference between two means. 4

(c) Obtain the test statistic for testing the significance of an observed sample correlation coefficient from a bivariate normal population. 3

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